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BOOK REVIEWS

Book Review Editor: Walter Van Assche

Books

A. Bultheel and M. Van Barel, *Linear Algebra*, *Rational Approximation and Orthogonal Polynomials*, Studies in Computational Mathematics **6**, North-Holland/Elsevier, Amsterdam, 1997, xvii + 446 pp.

This book is a nice example of the interaction between different areas of mathematics. Matrix analysis and numerical linear algebra are used in order to give a general framework for some aspects of approximation theory (orthogonal polynomials, continued fractions, and Padé approximants) where the algorithmic part plays a key role, as well as in applied domains like linear systems where the stability and realization theory need a substantial mathematical background.

Chapter 1 is devoted to the classical algorithm of Euclid with a more abstract formulation in a Euclidean domain. A matrix version of it, as well as the interpretation of formal continued fractions, gives a suggestive presentation of the approximation of formal Laurent series. Chapters 2 and 3 are focussed on some properties of Hankel matrices, in particular their factorization by the Schur algorithm, and the Lanczos algorithm for nonsymmetric matrices. Here, the breakdown problems are analyzed in detail and the interpretation in terms of biorthogonality is given in a very elegant way. Chapter 4 is, in my opinion, one of the most interesting contributions of the authors. Orthogonal polynomials are introduced as a translation of the above results to a general biorthogonal form with an arbitrary moment matrix. Block orthogonal polynomials are carefully analyzed when the moment matrix has singular principal submatrices. In particular, for the Hankel and Toeplitz moment matrices, numerical aspects like inversion algorithms are studied. They can be interpreted by using the generalized Christoffel–Darboux formulas. The last section of this chapter is devoted to formal orthogonality on algebraic curves. A more comprehensive and detailed presentation of this subject would be welcome because the authors basically reproduce a paper by Brezinski without any criticism: the connection with root location problems could be explored more. In Chapter 5, which deals with Padé approximation, the authors emphasize the computation of diagonal and antidiagonal Padé approximants as well as the minimal Padé approximation and its numerical implementation. Chapter 6 is my second favorite chapter of this book. It includes an excellent presentation of the minimal partial realization problem as well as a survey of recent developments on stability tests of the Routh–Hurwitz type (for continuous linear systems) and the Schur–Cohn–Jury type (for discrete linear systems). Chapters 7 and 8 give some perspectives on further developments like the general rational interpolation problems and an application of the Euclidean algorithm in the set of Laurent polynomials to the factorization of a polyphase matrix into a product of elementary continued fraction-like matrices. These are very useful in the computation of the wavelet transform and its inverse.

This book is very well written but it is not easy to read. It is, however, an important addition to the literature on the above mentioned topics. The presentation and an exhaustive set of updated references (243) give extra value to this work. In conclusion, I recommend it to

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anyone who is interested not only in approximation theory but also in its applications in engineering and computer science.

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M. L. Gorbachuk and V. I. Gorbachuk, *M. G. Krein's Lectures on Entire Operators*, Operator Theory: Advances and Applications **97**, Birkhäuser, Basel, 1997, x + 220 pp.

Mark Grigorievich Krein was one of the leading analysts from the former Soviet Union. His name will live on in the Krein–Milman theorem and the Adamyan–Arov–Krein theorem, to name but two of his many accomplishments.

The book under review is based on the notes taken by his students of Krein's lectures on entire operators delivered in Odessa in 1961. His theory on entire operators was developed in the 1940s, but a detailed account of it had not been published. Therefore, the present expanded version of the lecture notes is a very welcome addition to the literature.

The book is mainly about closed symmetric operators in a Hilbert space whose deficiency indices are $(1, 1)$. Such an operator A is called an entire operator if the following hold: First, there is for every complex number z a constant $\kappa_z > 0$ such that $\|(A - zI) f\| \geqslant \kappa_z \|f\|$ for all f in the domain of A; and second, that there exists a u in the Hilbert space which is outside the range of $A - zI$ for every $z \in \mathbb{C} \setminus \mathbb{R}$. Given such a u, there is an entire function $f_u(z)$, associated with every element f of the Hilbert space. This leads to an interesting interplay between operator theory and function theory. One of the results is that the map $f \mapsto f_u$ is an isometry from the Hilbert space to the space $L^2(\mathbb{R}, d\sigma)$ for some non-unique distribution function σ. Self-adjoint extensions of A correspond to certain discrete choices of σ.

As may be sensed from this description, the reader is expected to have a solid background in operator theory. The first two chapters cover the general theory, while the third chapter presents applications to the classical moment problem, positive definite functions, and unitary and spiral curves. There are two appendixes with extensions of the theory. Finally, there is a curious appendix with the notes taken from Krein's lecture at the jubilee session of the Moscow Mathematical Society in 1964, containing many interesting historical remarks.

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L. A. Sakhnovich, *Interpolation Theory and Its Applications*, Mathematics and Its Applications **428**, Kluwer, Dordrecht, 1997, xviii + 197 pp.

This monograph deals with the operator interpolation theory in a general setting. The author states the general interpolation problem for matrix (operator)-valued functions in terms of the so-called operator identities of the type

$$AS - SA^* = i(\Phi_1 \Phi_2^* + \Phi_2 \Phi_1^*). \tag{1}$$

Famous classical matrix interpolation problems are known to be equivalent to the identity (1) for some appropriate choice of the operators A, S, Φ_1, Φ_2. This is the case, for example, for the matrix Nevanlinna–Pick problem and the Hamburger matrix moment problem. At the same time the operator identity (1) is known to be connected with some operator inequalities, known as Potapov inequalities, which give necessary and sufficient conditions for the existence of a solution of the operator interpolation problem. This approach to the operator interpolation problems was first proposed by M. G. Krein and developed by the Ukrainian mathematical school. The author of the book is known as an active researcher in this area and this monograph presents the results of his investigations into these topics.

In Chapter 1 the author gives a necessary background for the operator identities, the operator inequalities, and the operator interpolation problems. The operator-valued functions $v(z)$ are supposed to have an integral representation with some positive definite matrix valued measure τ on the real line:

$$v(z)=\beta(z)+\alpha+\int_{-\infty}^{+\infty}\left(\frac{1}{u-z}-\frac{u}{1+u^2}\right)d\tau(u).$$

Chapter 2 deals with the same topics but in the case of measures τ with support on the unit circle, where the integral representation takes the form

$$F(z)=-i\alpha+\int_{-\pi}^{\pi}\frac{e^{i\phi}-z}{e^{i\phi}+z}\,d\tau(\phi).$$

In both cases it is shown that the interpolation problem is equivalent to some operator inequalities and the solution is given in the nondegenerate case.

Chapter 3 is concerned with the problem of the extension of Hermitian positive functions of several variables with the preservation of the Hermitian positiveness. This problem is well known in analysis. The classical results of Hilbert and Caldéron–Pepinski show that in many dimensions this extension is not always possible. The author gives a concrete example of this phenomenon. In Chapter 4 the author develops the operator inequality approach for studying de Branges spaces of entire functions and gives a new proof of the Parseval identity in this case.

Chapters 5 and 6 are devoted to the degenerate case of the interpolation problem and to some concrete problems. In particular, the matrix Nevanlinna–Pick problem and the Schur problem for the real line and for the unit circle are investigated in detail. In Chapter 7 the author studies the extremal solutions of the interpolation problems. The approach based on the operator identities is shown to be useful for solving these problems.

In Chapters 8–10 the author applies the operator identity method to study the spectral theory of canonical systems of difference operators and uses it for the investigation of the semi-infinite Toda lattice in two cases: with free and fixed end on the left. He also gives a solution of the finite lattice with fixed end.

The final chapter (Chapter 11) deals with the Nevanlinna class of functions with operator argument. As an application the author gives a generalization of the well known Sarason factorization theorem to operator arguments.

This book presents some aspect of general operator interpolation theory and may be interesting to the specialists in the field. The bibliography with 63 items seems to be selected by the taste of the author and is obviously incomplete. The book is sufficiently well organised but I can not recommend it for a first reading in these topics.

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P. Wojtaszczyk, *A Mathematical Introduction to Wavelets*, London Mathematical Society Student Texts **37**, Cambridge University Press, Cambridge, UK, 1997, xii + 261 pp.

This book presents a self-contained and comprehensive mathematical introduction to the theory of orthogonal wavelets and their uses in analysing functions and function spaces, both in one and in several variables. The contents of the book can be divided into two parts. The first part is an introduction to constructions of orthogonal wavelets (Chapters 1–4 introduce one-variable wavelets and Chapter 5 discusses multivariable generalizations). The second part (Chapters 6–9) deals with wavelet expansions and function spaces.

Chapter 1, entitled "A Small Sample," gives an overview of the book. Two pre-wavelets are discussed, the Haar wavelet and the Stromberg wavelet. Indeed, they were invented and investigated well before the notion of the wavelet appeared. The construction and properties of those wavelets are presented here as well as some sample theorems about convergence of wavelet expansions. Chapter 2 introduces the general theory. The concepts of multiresolution analysis and the scaling function are presented there. Then all wavelets associated with a given multiresolution analysis are described. Periodic wavelets are discussed here in general terms. Chapter 3 shows how the above general theory can be applied in concrete cases, e.g., to construct Meyer's wavelets and spline wavelets and to discuss in detail their smoothness and decay. Chapter 4 discusses wavelets with compact support. A general approach to constructing compactly supported wavelets is presented and applied to a construction of smooth, compactly supported wavelets. Chapter 5 discusses multivariable generalisations. Many examples of Haar-like multivariable wavelets are given, then more smooth examples are constructed.

Chapter 6 gives a self-contained presentation of the basic theory of L_p, H, and BMO spaces. Chapter 7 introduces unconditional convergence of series in Banach spaces and discusses the concept of unconditional basis. In Chapter 8 it is proven that wavelets provide unconditional bases in L_p and H spaces. Also, a characterization of those functional spaces in terms of wavelet expansions is given. Finally, Chapter 9 discusses moduli of continuity and Besov norms and their connection with wavelets.

Each chapter ends with a section called "Sources and Comments" and with exercises. The exercises range from quite easy and routine to rather difficult. Overall, this is a valuable textbook for those wishing to learn more about the mathematical foundation of wavelets.

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George E. Andrews, Richard Askey, and Ranjan Roy, *Special Functions*, Encyclopedia of Mathematics and Its Applications **71**, Cambridge University Press, Cambridge, UK, 1999, 664 pp.

Special functions are useful in engineering, applied science, numerical analysis, and computer science, to say the least. They are also useful in the study of mathematics. The present book is not just another more or less encyclopedic collection of tables and formulas, but an ordered treatise on the subject. The aim of the book is to show how research was, and is being performed in this field. We see how people like Euler, Gauss, Jacobi (called "the great algorist" by E.T. Bell), and Weierstrass felt the importance of special functions, and how this study is still important as a source of mathematical knowledge.

The following is a short list of the chapters' contents:

1. The gamma and beta functions and the main results, and also the zeta functions, p-adic and finite-field analogs, and summation formulas.

2 and 3. Hypergeometric functions and identities, with transformations; the beautiful and deep Riemann theory of the differential equation; the Barnes theory (and not just the Barnes formula); dilogarithms; the arithmetic–geometric mean; the Wilf–Zeilberger method; and the problem of "human thought aided by machines" (p. 176).

4. Bessel functions and confluent hypergeometric functions, with, a.o., properties of zeros (reality, monotonicity).

5, 6, and 7. Orthogonal polynomials and the general theory, with Gauss quadrature, continued fractions, completeness, special (i.e., hypergeometric) orthogonal polynomials, an extended treatment of connection and linearization coefficients, and related results: proof of the Bieberbach conjecture and the irrationality of $\zeta(3)$.

8. The Selberg integral and its applications, the discriminants of classical orthogonal polynomials and the equilibrium of many-particle systems.

9. Spherical harmonics and representation of rotation groups.

10. An introduction to q-series, the q-analogs of Chapter 1, with three proofs of Ramanujan's summation formula (a q-analog of the integral form of the beta function), theta functions, and q-ultraspherical polynomials.

11. Partitions.

12. Bailey chains, the heart of the most striking findings of Rogers, Ramanujan, Bailey, and others, as well as of the authors, and exercises on q-classical orthogonal polynomials.

Appendixes (1) Infinite products; (2) Summability and fractional integration; (3) Asymptotic expansions; (4) Euler–Maclaurin summation formula; (5) Lagrange inversion formula; and (6) Series solutions of differential equations.

Each chapter ends with exercises which are often deep complementary remarks and also with unexpected results from the vast literature known to the authors (including a comic strip—see pp. 60 and 641).

As one may expect, there are very few omissions and misprints. There could have been a comment on the singularity at $x=1$ of the logarithmic integral $\mathrm{li}(x)$ treated on p. 197; the mysterious "N. G. de Bruin" should be "de Bruin, M. G." in the 12th item (date 1981) of the references list (p. 644) and in the reference to p. 234 in the index (p. 655), and "de Bruijn, N. G." in the 11th item (date 1967) of the references list (p. 644) and in the reference to p. 282 in the index (p. 655).

This is an extremely valuable book for engineers, applied scientists, numerical analysts, and computer scientists, to say the least, *and* for mathematicians.

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P. Deift, *Orthogonal Polynomials and Random Matrices*: *A Riemann–Hilbert Approach*, Courant Lecture Notes in Mathematics **3**, Courant Institute of Mathematical Sciences, New York, 1999, 273 pp.

Recently a group of scientists formed around Percy Deift has introduced a new tool in the theory of orthogonal polynomials: they connect the asymptotic behavior of orthogonal polynomials with a Riemann–Hilbert problem. This is a very successful new tool and several interesting papers using this approach have been published already. This is one of the reasons why Percy Deift (together with Xin Zhou and Peter Sarnak) received the George Pólya prize from SIAM in 1998.

The book under review is based on a course given by Deift at the Courant Institute in 1996/1997. As such it gives a very welcome introduction to aspects of the Riemann–Hilbert problem, radom matrices, and orthogonal polynomials for graduate students and more advanced researchers, especially those who are interested in the interaction between orthogonal polynomials, random matrices, Riemann–Hilbert problems, continued fractions, and equilibrium problems in logarithmic potential theory.

The lecture notes start by explaining what a Riemann–Hilbert problem is, and many examples are given where Riemann–Hilbert problems arise: the nonlinear Schrödinger equation, the Korteweg–de Vries equation, the Boussinesq equation, Burger's equation, Toda equations, etc. Chapter 2 ("Jacobi Operators") and Chapter 3 ("Orthogonal Polynomials") deal with some basic results in the theory of orthogonal polynomials, with emphasis on the corresponding Jacobi matrices and operators. It is explained how the spectrum and the spectral measure of a Jacobi operator are connected with the orthogonality measure (and its support) of orthogonal polynomials through the spectral theorem. Both bounded and unbounded Jacobi operators are considered. Jacobi operators turn up in the Lax-pair for the Toda flow and it is left as an exercise to the reader to solve the Toda flow using an isospectral deformation of the spectral measure of a Jacobi operator. In the chapter on orthogonal polynomials the author emphasizes the Hankel matrix containing the moments and a formula expressing the Hankel determinant as an n-fold integral of "partition function" nature. Following Fokas, Its, and Kitaev it is shown how one can obtain the orthogonal polynomials from their orthogonalizing measure via a Riemann–Hilbert problem. Orthogonal polynomials and the Riemann–Hilbert approach to obtaining their asymptotic behavior are taken up later in Chapter 7, which contains the recent work of Deift and his collaborators on this subject.

Chapter 4 is on continued fractions, historically the origin of orthogonal polynomials. The emphasis here is on the continued fraction expansion of a number. The author explains Aryabhata's approach of obtaining integer solutions x, y of the Diophantine equation $ax \pm by = c$, where a, b, c are integers. The continued fraction expansion of a number also leads to some interesting problems in measure theory and ergodic theory, which are clearly reviewed. Naturally, the connection with Jacobi operators and orthogonal polynomials is made.

Another important subject treated in these lecture notes are random matrices. Indeed, in the preface the author mentions that the course was given in an attempt to understand the various results and formulas in Mehta's book *Random Matrices* from a more rigorous mathematical point of view. Chapter 5 deals with random matrix theory. It is a well written and clear introduction to the subject and explains unitary ensembles, spectral variables for Hermitian matrices, and the distribution of eigenvalues and spacings of eigenvalues, and ends with a formulation of the various universality problems for the unitary ensembles of Hermitian matrices, which is taken up again in Chapter 8 where universality is proved rigorously using the connection with orthogonal polynomials. Indeed, the probability distribution of the eigenvalues is of the same form as the n-fold integral for the Hankel determinant of certain orthogonal polynomials.

Chapter 6 gives the mathematical tools for the investigation of the distribution of the eigenvalues of random matrices *and* the distribution of zeros of orthogonal polynomials. These eigenvalues and zeros usually are unbounded so that a proper scaling is needed. To obtain the scaling and the distribution of the scaled eigenvalues and zeros, one uses logarithmic potential theory with external fields. Again, this is clearly explained for an audience with minimal background.

In conclusion I can state that I strongly recommend this as a very good textbook for a graduate course, containing all the material needed to understand how the Riemann–Hilbert approach indeed gives a very useful tool in the analysis of orthogonal polynomials and random matrices.

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M. Petkovšek, H. Wilf, and D. Zeilberger, $A = B$, A. K. Peeters, Wellesley, MA, 1998, xii + 221 pp.

This remarkable book is extremely well-written and gives a complete selfcontained exposition of a fascinating breakthrough in the field of computer algebra and automatic theorem proving. It's about computer programs for simplifying sums that involve binomial coefficients and for discovery and proof of hypergeometric identities. The authors of the book played key roles in these exciting new developments.

Let n and k be variables running over $\mathbb{Z}$. A sequence t_k is called a hypergeometric term if t_{k+1}/t_k is a rational function of k. A double sequence $F(n, k)$ is called a hypergeometric term if both $F(n+1, k)/F(n, k)$ and $F(n, k+1)/F(n, k)$ are rational functions of n and k. The basic problem considered in this book is the development of algorithms to discover and prove closed formulas for sums of the form $s_n := \sum_{k=0}^{n-1} t_k$ and $f(n) := \sum_{k \in \mathbb{Z}} F(n, k)$. We always assume that t_k and $F(n, k)$ are hypergeometric terms and that for each n there are only finitely many k with $F(n, k) \neq 0$. As an example we only mention the classical identities

$$\sum_k \binom{n}{k}^2 = \binom{2n}{n} \quad \text{and} \quad \sum_k (-1)^k \binom{2n}{k}^3 = (-1)^n \, (3n)!/(n!)^3, \tag{1}$$

referring the reader to the book for dozens of other examples. Such identities appear very often and are very important in combinatorics and discrete mathematics.

The first three chapters of the book are introductory: some nice classical examples of automatic identity proving are given in the context of trigonometric and elliptic functions, just to warm up the reader, and afterward hypergeometric terms and functions are introduced with many examples.

The equality of two sequences can be proved by showing that both sequences satisfy the same recurrence relation and have the same initial conditions. Hence it is essential to find recurrences for sums of the form $\sum_k F(n, k)$. The first automatic way to achieve this was discovered by Sister Celine Fasenmyer in her Ph.D. thesis in 1945. This is the content of Chapter 4 of the book. Fasenmyer proved that $F(n, k)$ always satisfies a nontrivial recurrence relation

$$\sum_{i=0}^{I} \sum_{j=0}^{J} a_{i,j}(n) \, F(n-j, k-i) = 0, \tag{2}$$

where I, J are suitable positive integers and $a_{i,j}(n)$ are polynomials in n. Such a recurrence can be obtained as follows: divide all terms of (2) by $F(n, k)$ and clear the denominators of the rational functions $F(n-j, k-i)/F(n, k)$. In this way the left-hand side of (2) becomes a polynomial in k, and equating its coefficients to zero gives a linear system of equations in the coefficients of the polynomials $a_{i,j}(n)$, which is solvable when I, J are large enough. Clearly (2) implies the recurrence $\sum_{j=0}^{J} b_j(n) f(n-j)$, for $f(n) := \sum_k F(n, k)$, where $b_j(n) = \sum_{i=0}^{I} a_{i,j}(n)$.

Chapter 5 treats Gosper's algorithm, discovered in 1978, which answers the following question: given a hypergeometric term t_n, is there a hypergeometric term z_n satisfying $z_{n+1} - z_n = t_n$? If the answer is affirmative, then $s_n := \sum_{k=0}^{n-1} t_k$ equals z_n plus a constant, and the algorithm outputs a closed formula for s_n. In that case t_n is called Gosper-summable. The question is clearly equivalent with the question of whether the recurrence $y_{n+1} r_n - y_n = 1$ has a rational function solution y_n, where r_n is the rational function t_{n+1}/t_n. Indeed, if z_n is hypergeometric, then $y_n := z_n/t_n = 1/(z_{n+1}/z_n - 1)$ is rational and satisfies the recurrence. It is this reduction of the question which is the key to Gosper's algorithm.

Chapter 6 is about Zeilberger's algorithm, also called the method of creative telescoping. We saw already that Fasenmyer's method gives a recurrence for $f(n) := \sum_k F(n,k)$, but actually this algorithm is too slow. It was Zeilberger's idea that in order to find a recurrence for $f(n)$ it suffices to find a nontrivial recurrence of the form

$$\sum_{j=0}^{J} a_j(n)\, F(n+j,k) = G(n,k+1) - G(n,k), \tag{3}$$

where the $a_j(n)$ are polynomials in n and $G(n,k)$ is a suitable hypergeometric term, which is necessarily of the form $G(n,k) = R(n,k)\, F(n,k)$ with $R(n,k)$ a rational function of n and k. Note that (3) immediately implies the recurrence $\sum_{j=0}^{J} a_j(n) f(n+j) = 0$ for $f(n)$. The existence of a recurrence of the form (3) is an easy consequence of Fasenmyer's recurrence (2). Zeilberger's outstanding contribution (1990) was finding a much faster algorithm for obtaining (3). His method is to determine the coefficients of the polynomials $a_j(n)$ by requiring that the left side of (3) be Gosper-summable. Indeed, a delicate analysis of Gosper's method yields linear equations in the coefficients of the $a_j(n)$'s and some extra variables. One first tries this with $J=1$, $J=2$, etc. When $J=1$, $f(n)$ is clearly hypergeometric and one obtains a closed formula for $f(n)$. For example, if we apply this to the second sum in (1), we get $J=1$, $a_0 = 3(3n+2)(3n+1)$, and $a_1 = (n+1)^2$, from which one deduces directly the second identity in (1).

Chapter 7 deals with the WZ phenomenon, due to Wilf and Zeilberger (1990). This yields an amazingly short method for certifying the truth of "most" identities of the form $\sum_k F(n,k) = 1$. Wilf and Zeilberger observed that for "most" such identities (say, 99% of the time) if we apply Zeilberger's algorithm we get a recurrence (3) of a very special form, namely

$$F(n+1,k) - F(n,k) = G(n,k+1) - G(n,k). \tag{4}$$

Note that (4) implies the identity $\sum_k F(n,k) = 1$, whenever this identity holds for $n=1$. In such a case we have an extremely succinct proof of the identity. A WZ-proof certification then consists of giving a rational function $R(n,k)$ such that $G(n,k) := R(n,k)\, F(n,k)$ satisfies (4). If the computer provides us with such a certificate, then we can often verify (4), and the identity for $n=1$, by hand. For example, the first identity in (1) is equivalent with the identity $\sum_k \binom{n}{k}^2/\binom{2n}{n} = 1$, for which $R(n,k) = -k^2(3n-2k+3)/2\,(2n+1)(n-k+1)^2$. In the same chapter it is also explained how to discover new identities whenever one has a WZ-proof certificate for a known identity.

Chapter 8 treats the algorithm of Petkovšek (1991) which, given a linear recurrence with polynomial coefficients, finds all solutions which can be written as a finite linear combination of hypergeometric terms. Combined with Zeilberger's algorithm this gives a method for determining whether $\sum_k F(n,k)$ can be written as a finite linear combination of hypergeometric terms and for finding the explicit formula whenever it exists. Finally, Chapter 9 uses operator algebra methods to find a recurrence for $\sum_k F(n,k)$ when $F(n,k)$ is not hypergeometric but holonomic (meaning that $F(n,k)$ satisfies "enough" recurrence relations both in n and k).

Although I considered only one-variable sums in this review, the book also deals with multivariate sums and also treats q-generalizations. The book is written in an exceptionally clear way and can be read by anyone who has had at least one year of university mathematics. The many examples, all verifiable in real time on your PC, make the book very lively. The material is absolutely fascinating, both for undergraduates and for professional mathematicians. Maple and Mathematica programs that implement the algorithms described in the book can be found on the World Wide Web at the site http://www.math.temple.edu/ ~ zeilberg. Moreover, you can download the entire book from this URL!

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Proceedings

Approximation Theory and Optimization, M. D. Buhmann and A. Iserles, Eds., Cambridge University Press, Cambridge, UK, 1997, xiii + 220 pp.

On the occasion of the 60th birthday of M. J. D. Powell, a conference on numerical mathematics was held in Cambridge, UK, on July 27–30, 1996. The present volume contains 10 invited papers presented at this conference and a description of M. J. D. Powell's contributions to numerical mathematics, with a brief review of his work in univariate and multivariate approximation, but also of his contributions to optimization. There is a picture of M. J. D. Powell and a list of his publications.

Approximation Theory IX, Charles K. Chui and Larry L. Schumaker, Eds., Vol. I; *Theoretical Aspects*, and Vol. II, *Computational Aspects*, Vanderbilt University Press, Nashville/London, 1998, xvii + 354 pp. (Vol. I) and xvii + 392 pp (Vol. II).

There have been eight international conferences on approximation theory held in Texas (Austin and College Station) since 1973, approximately once every two or three years. The conference proceedings were always a useful source of information. The Ninth International Conference on Approximation Theory was hosted by Vanderbilt University in Nashville, TN, January 3–6, 1998. The conference was attended by 190 mathematicians from 21 countries. The proceedings are published in two separate volumes: the first volume covers theoretical aspects from several areas of classical and modern approximation theory (40 contributions) and the second volume is devoted to recent applied and computational developments (40 contributions). Among the contributions are the invited survey talks, the talk of the most recent Popov Prize winner Arno Kuijlaars (Vol. I, pp. 201–221) and various papers from the eight special sessions on wavelets, approximation in the complex plane, radial basis functions, abstract approximation, neural networks, splines, computer aided geometrical design, and non-linear m-term approximation.

Special Functions and Differential Equations, K. Srinivasa Rao, R. Jagannathan, G. Vanden Berghe, and J. Van der Jeugt, Eds., Allied, New Delhi, 1998, xii + 486 pp.

These are the proceedings of a workshop devoted to *special functions and their q-generalizations* and *differential equations and numerical methods* which was held at the Institute of Mathematical Sciences in Madras (now known as Chennai), India, January 13–24, 1997. There is a section on special functions with 39 contributions and a section on differential equations with 7 contributions.

Orthogonal Functions, Moment Theory, and Continued Fractions, W. B. Jones and A. Sri Ranga, Eds., Lecture Notes in Pure and Applied Mathematics **199**, Dekker, New York, 1998, xii + 416 pp.

This volume contains the proceedings of a research conference (workshop) held at the Hotel Fazenda *Solar das Andorinhas* in Campinas, Brazil, June 19–28, 1996. The conference talks dealt with topics from orthogonal functions, moment theory, continued fractions, Padé approximants, Gaussian quadrature, Stieltjes transforms, and linear functionals. The volume contains 21 refereed contributions.